CHARACTERIZATION AND COMPRESSION OF TURBULENT SIGNALS AND IMAGES USING WAVELET-PACKETS

Lareef Zubair, K.R. Sreenivasan & M. Victor Wickerhauser

Yale University
New Haven, CT 06520

ABSTRACT

The newly introduced Wavelet-Packet transform (Coifman & Meyer 1989, Coifman et al. 1990) allows the decomposition of a signal as a function of the scale, the position and the frequency (or wavelength) optimally. Each Wavelet-Packet coefficient provides insight into the structure of the data locally and at the appropriate scale. We have applied this transform technique to one-dimensional data and two-dimensional images and report on its ability to characterize turbulence data with a few coefficients. We find that, overall, the Wavelet-Packet transform technique performs better than its competitors. That is, significant data compression ratios can be achieved without severely distorting the signal or the image.

1. Introduction

Turbulent motion has traditionally been decomposed in terms of Fourier modes, and one speaks of its frequency or wavenumber components. Our intuition about 'large' and 'small' scales of motion is substantially biased by Fourier description. There are several reasons why one should think of alternative descriptions. First, most turbulent flows (except for the hypothetical case of infinitely extended homogeneous turbulence) are finite in spatial extent, at least in one or

T. G. Gatski et al. (eds.), *Studies in Turbulence*

two directions, so that expansions in terms of Fourier modes depend on mode-cancellation outside the flow domain – this being uneconomical in general. One therefore needs basis functions with compact support in real space. Secondly, a description of sharp gradients by Fourier modes is inefficient. Finally, turbulent flows (even the homogeneous ones) possess spatial structure of some sort, and the question of how to represent them efficiently is gaining increasing importance. The structure is stronger in some flows than in others and has better definition at some scales than at others, but it is clear that one should make a distinction between Fourier modes on the one hand and spatial structures on the other. Indeed, descriptive efforts in turbulence have always resorted to a variety of 'turbulent eddies' (see for example, Townsend 1956, 1976). Whatever the precise meaning of the term 'eddy', *it is clear that an eddy is not a Fourier mode*!

Lumley (Tennekes & Lumley 1972) recognized this distinction and wrote as follows (p.259): 'The Fourier transform of a velocity field is a decomposition into waves of different wavelengths; each wave is associated with a single Fourier coefficient. An eddy, however, is associated with many Fourier coefficients and the phase relations among them. Fourier transforms are used because they are convenient (spectra can be measured easily); more sophisticated transforms are needed if one wants to decompose a velocity field into eddies instead of waves ...' In particular, figure 1 shows Lumley's schematic of an eddy in both real and Fourier spaces. It turns out that this eddy, which has the desired property of being spatially compact, is an example of what are now known as 'wavelets'. It is very much to Lumley's credit that he should have introduced wavelets to turbulence, albeit without using the name, at least as early as 1972! Yet, the broad recognition that wavelets possess useful mathematical properties that are appropriate for turbulence description is quite recent.

Formally, a wavelet is a spatially localized function which can be translated and rescaled while maintaining its shape (see, for example, Daubechies 1988). Wavelets in general are not local in Fourier space, and there is the so-called 'uncertainty relation' which tells us that spatial localization and wavenumber localization are complementary, and that there is a quantifiable trade off between them. Wavelet analysis provides a means for studying scaling and transient behavior of signals by using the dilates and translates of the wavelet as a basis. Decomposition of the signal into these basis